VAISHALI EDUCATION POINT

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RELATIONS & FUNCTIONS

Class :- XII **Subject :- MATH**

QNo.	Questions
1	Show that the relation R in the set R of real numbers, defined as R = {(a, b): $a \le b^2$} is neither reflexive nor symmetric nor transitive.
2	Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.
3	Show that the relation R in R defined as R = {(a, b): $a \le b$}, is reflexive and transitive but not symmetric.
4	Check whether the relation R in R defined as R = {(a, b): $a \le b^3$} is reflexive, symmetric or transitive.
5	Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.
6	Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x and y have same number of pages} is an equivalence relation.
7	Show that the relation R in the set A = {1, 2, 3, 4, 5} given by , is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of 2, 4}.
8	Show that each of the relation R in the set A = {$x \in Z : 0 \le x \le 12$ }, given by (i) R = {(a, b) : | a – b | is a multiple of 4} (ii) R = {(a, b) : a = b} is an equivalence relation. Find the set of all elements related to 1 in each case.
9	Given an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric.
10	Show that the relation R in the set A of points in a plane given by R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all point related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.
11	Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2): T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, $T_2$and T_3 are related?
12	Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2): P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?
13	Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L_1, L_2): L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.
14	Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer. (A) R is reflexive and symmetric but not transitive. (B) R is reflexive and transitive but not symmetric. (C) R is symmetric and transitive but not reflexive. (D) R is an equivalence relation.
15	Let R be the relation in the set N given by R = {(a, b): a = b − 2, b > 6}. Choose the correct answer. (A) (2, 4) ∈ R (B) (3, 8) ∈R (C) (6, 8) ∈R (D) (8, 7) ∈ R
16	Show that the function f: $R_* \to R_*$ defined by $f(x)=\frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_*?
17	Check the injectivity and surjectivity of the following functions: (i) f: N → N given by f(x) = x^2 (ii) f: Z → Z given by f(x) = x^2 (iii) f: R → R given by f(x) = x^2 (iv) f: N → N given by f(x) = x^3 (v) f: Z → Z given by f(x) = x^3
18	Prove that the Greatest Integer Function f: R → R given by f(x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.
19	Show that the Signum Function f: R → R, given by $f(x)=\begin{cases}1, & \text{if } x>0\\0, & \text{if } x=0\\-1, & \text{if } x<0\end{cases}$ is neither one-one nor onto.
20	Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.
21	In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer. (i) f: R → R defined by f(x) = 3 − 4x (ii) f: R → R defined by f(x) = 1 + x^2
22	Show that the binary operation * defined by $a * b = ab + 1$ is not associative. **(2007)**

23	$f: R \to R$ defined as $f(x) = \frac{2x-7}{4}$ is an invertible function, find f^{-1} **(2008 Comp.)**
24	Let * be a binary operation on the set Q of rational numbers given as $a * b = (2a-b)^2, a, b \in Q$. Find $3*5$ and $5*3$. Is 3 * 5 = 5 * 3? **(2008 Comp.)**
25	(i) Is the binary operation *, defined on set N, given by $a * b = \frac{a+b}{2}$ for all $a, b \in Q$, commutative ? (ii) Is the above binary operation * associative? **(2008)**
26	Let * be a binary operation on set Q of rational numbers defined as $a * b = \frac{ab}{5}$ Write the identity for *, if any. **(2009)**
27	If the function $f: R \to R$ is given by $f(x) = \frac{x+3}{3}$ and $g: R \to R$ is given by $g(x) = 2x - 3$ find (i) fog and (ii) gof. Is $f^{-1} = g$? **(2009 Comp.)**
28	If the binary operation * on the set of integers Z, is defined by $a * b = a + 3b^2$, then find the value 2 * 4. **(2009)**
29	Prove that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : \|a - b\| \text{ is even}\}$, is an equivalence relation. **(2009)**
30	If $f: R \to R$ is defined by $f(x) = 3x + 2$, find $f(f(x))$ **(2010 Comp.)**
31	Show that the function $f: R \to R$ given by $f(x) = ax + b$, where $a, b \in R, a \neq 0$ is a bijective function. **(2010 Comp.)**
32	What is the range of the function $f(x) = \frac{\|x-1\|}{(x-1)}$? **(2010)**
33	Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b) : a, b, \in Z, \text{ and } (a - b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation. **(2010)**
34	Let * be a binary operation on set of integers I, defined by $a * b = 2a + b - 3$. Find the value of 3 * 4 **(2011 Comp.)**
35	If $f: R \to R$ be the function defined by $f(x) = 4x^3 + 7$, show that f is a bijective function **(2011 Comp.)**
36	State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive **(2011)**
37	Consider the binary operation * on the set $\{1, 2, 3, 4, 5\}$ defined by a * b = min {a, b}. Write the operation table of the operation*. **(2011)**
38	If f(x) is an invertible functions, find the inverse of $f(x) = \frac{3x-2}{5}$.
39	Let T be the set of all triangles in a plane with R a relation in T given by R = {(T_1, T_2): T_1 is congruent to T_2}. Show that R is an equivalence relation.
40	Consider the division of set A = {1, 2, 3, 4, 5, 6, 7, 8} by subsets {1, 6}, {2, 7}, {3, 8}, {4}, {5}. Define a relation in A by R = {(a, b)} : a and b in the same subset of the division of A}. Show that R is an equivalence relations.
41	Consider the binary operation $^*R \times R \to R$ and $0 : R \to R$ Defined as $a ^* b = \|a - b\|$ and $a o b = \forall\ a, b, \in R$. Show that $a^*(b o c) = (a ^* b) o (a ^* c), \forall a$.
42	Show that if f : A → B and g : B → C are one-one, then go f : A → C is also one-one.
43	Let S = {1, 2, 3}. Find whether the function f : S → S defined as f = {(1, 3), (3, 2), (2, 1)} has inverse. If yes, find f^{-1}.
44	Let f, g, h be the real valued functions. Show that (f + g) oh = foh + goh.
45	Find gof and fog, if f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$.
46	Show that f : {–1, 1} → R, given by $f(x) = \frac{x}{x+2}, x \neq -2$, is one-one. Find the inverse of function $f : [-1, 1] \to R_f$.
47	Consider the function f : {1, 2, 3} → {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. Find $(f^{-1})^{-1}$ = f.
48	Show that the number of equivalence relation in the set {1, 2, 3} containing (1, 2) and (2, 1) is two.
49	If f(x) = e^x and g(x) = log x, show that fog = gof, x > 0.
50	If $f(x) = \frac{5x+3}{4x-5}, x \neq \frac{5}{4}$, show that fof is an identity function.
51	Prove that the function f : Q → Q, given by f(x) = 2x – 3, is objective.

52	If f(x) = sin x, g(x) = cos x and h(x) = 2x. 'f', 'g', 'h' being real valued functions, show that ho(fg) = foh.
53	Let * be a binary operation on N, given by a * b = l.c.m. (a, b) for a, b ∈ N. Find : (i) 2 * 4, (ii) 3 * 5, (iii) Is '*' associative?
54	Let A = N × N and let * be a binary operation on A defined by (a, b) * (c, d) = (ac, bd). Show that (i) (A, *) is commutative (ii) (A, *) is associative. Find the identify element, if any , in A.
55	Show that f : N × N given by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x+1, \text{ if } x \text{ is even} \end{cases}$ is both one-one and onto.
56	Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y)\ R\ (u, v) \Leftrightarrow xv = yu$. Show that R is an equivalence relation.
57	Define the binary operation '*' on set A = {0, 1, 2, 3, 4, 5} given by a * b = (ab) mod 6 {(ab) mod 6, means remainder obtained after dividing ab by 6}. Show that 1 is the identity for '*', 1 and 5 are the only invertible elements with $1^{-1} = 1$ and $5^{-1} = 5$.
58	A relation R : N × N is given by R = {(a, b) : b is divisible by a}. Check whether R is an equivalence relation.
59	Show that the relation R : N × N defined by $(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ for all (a, b), (c, d) ∈ N × N is an equivalence relation.
60	Consider function $f : \left[0, \frac{\pi}{2}\right] \to R$ given by f(x) = sin x and $g : \left[0, \frac{\pi}{2}\right] \to R$ given by g(x) = cos x. Show that 'f' and 'g' are one-one but 'f + g' is not one-one.
61	Define a binary operation * on the set A = {0, 1, 2, 3, 4, 5} as $a * b = \begin{cases} a+b, if\ a+b < 6 \\ a+b-6, if\ a+b \geq 6 \end{cases}$. Show that 0 is the identity for this operation and each element a of the set is invertible with 6 – a being the inverse of a.
62	Let * be a binary operation defined on Q. Find which of the binary operation are associative. (i) a * b = a – b (ii) $a * b = \frac{ab}{4}$ (iii) a * b = a – b + ab (iv) a * b = ab^2.
63	Relation R in human beings in a city at a particular time is defined as R = {(a, b) : a likes b}. Is R an equivalence relation?
64	If f(x) is an invertible functions, find the inverse of $f(x) = \frac{3x-2}{5}$.
65	If f(x) = x + 7 and g(x) = x – 7, x ∈ R, find (fog) (7).
66	Let A be the set of all the students of a boy's school. Let relation R in set A is given by R = {(a, b) ∈ A × A : a is sister of b}. Can we say that R is an empty relation? Give reasons.
67	Show that the relation R in the set {1, 2, 3}, given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3) is reflexive but not symmetric.
68	Show that the function f : N → N given by f(x) = 2x, is one-one but not onto.
69	Show that the function f : N → N given by f(1) = f(2) = 1 and f(x) = x – 1 for x > 2, is not but not one-one.
70	Show that subtraction is not a binary operation on N.
71	Show that the operation * : R → R given by a * b → a + $4b^2$ is a binary operation.
72	Show that the binary operation * : R → R given by a * b = a + 2b is not commutative.
73	Let 'f' be the exponential function and 'g' be logarithmic function, find (f + g) (1).
74	Let binary operation * : Q → Q be defined as a * b = a – b + ab . Is '*' associative?
75	Let f : N → defined by f(x) = 3x. Show that 'f' is not an onto function.
76	Let 'f' and 'g' be two real functions defined as f(x) = 2x –3; g(x) = $\frac{3+x}{2}$. Find fog and gof. Can you say one is inverse of the other?
77	Is sin function onto in the set of real numbers? Explain with example.
78	Let f : R → R is defined by f(x) = |x|, Is function f onto ? Give reasons.
79	Let T be the set of all triangles in a plane with R a relation in T given by R = {(T_1, T_2): T_1 is congruent to T_2}. Show that R is an equivalence relation.
80	Consider the division of set A = {1, 2, 3, 4, 5, 6, 7, 8} by subsets {1, 6}, {2, 7}, {3, 8}, {4}, {5}. Define a relation in A by R = {(a, b)} : a and b in the same subset of the division of A}. Show that R is an equivalence relations.