VAISHALI EDUCATION POINT

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INTEGRALS

Class :- XII
SUBJECT MATHS

General Instructions

QNo.	Questions
1	$\int \frac{e^{x-1}+e^{e-1}}{e^x+x^e}\,dx$ **Ans.** $\frac{1}{e}\log(e^x+x^e)$
2	$\int \frac{1+\cot x}{x+\log\sin x}\,dx$ **Ans.** $\log\lvert x+\log(\sin x)\rvert + C$
3	$\int e^{5\log x}(x)^4\,dx$ **Ans.** $\frac{x^{10}}{10}+C$
4	$\int \frac{1}{e^x-1}\,dx$ **Ans.** $\log\left(\frac{e^x-1}{e^x}\right)+C$
5	$\int \frac{1}{\sqrt{x}+x}\,dx$ **Ans.** $2\log\lvert 1+\sqrt{x}\rvert + C$
6	$\int_0^{25}[x]\,dx$ **Ans.** 2
7	$\int_0^{\frac{\pi}{2}}\log(\tan x)\,dx$ **Ans.** 0
8	$\int_0^{\frac{1}{2}}\frac{1}{2+8x^2}\,dx$ **Ans.** $\pi/4$
9	$\int_{-\pi/4}^{\pi/4} x^3\sin^4 x\,dx$ **Ans.** 0
10	$\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx$

Ans. 0

11 $\int_{-\pi/4}^{\pi/4} |\sin x| dx$

Ans. $2-\sqrt{2}$

12 $\int_{0}^{1} x(1-x)^3 dx$

Ans. [1/20]

13 $\int \frac{e^{\sqrt{x}} \cos e^{\sqrt{x}}}{\sqrt{x}} dx$

Ans. $2 \sin e^{\sqrt{x}} + C$

14 $\int \frac{x^2}{16+25x^6} dx$

Ans. $\frac{1}{60} \tan^{-1} \frac{5x^3}{4} + C$

15 $\int \frac{e^x (1+x)}{\sin^2 (xe^x)} dx$

Ans. $-\cot (xe^x) + C$

16 $\int \frac{\sec x \operatorname{cosec} x}{\log \tan x} dx$

Ans. $\log |\log (\tan x)| + C$

17 $\int \frac{1}{\cos^2 x (1-\tan x)^2} dx$

Ans. $\frac{1}{1-\tan x} + C$

18 $\int \tan^3 x \, dx$

Ans. $\frac{\tan^2 x}{2} + \log \cos x + C$

19 $\int \frac{1}{\sin x + \cos x} dx$

Ans. $\frac{1}{\sqrt{2}} \log \tan \left(\frac{\pi}{2} + \frac{\pi}{8} \right) + C$

20 $\int \frac{1+\cos x}{\sin x \cos x} dx$

Ans. $\log \left| \tan . \tan \frac{x}{2} \right| + C$

21 $\int \frac{1}{\cos (x+a) \sin (x+b)} dx$

Ans. $\frac{\log \sin (x+b) \sec (x+a)}{\cos (a-b)} + C$

22 $\int \frac{1}{x^2+4x-5}\,dx$

Ans. $\frac{1}{6}\log\frac{x-1}{x+5}+C$

23 $\int \sqrt{x^2+4x-5}\,dx$

Ans. $\frac{1}{2}(x+2)\sqrt{(x+2)^2-3^2}-\frac{3^2}{2}\log\left|(x+2)+\sqrt{(x+2)^2-3^2}\right.$

24 $\int \frac{1}{1-\sin^4 x}\,dx$

Ans. $\frac{1}{2}\tan x+\frac{1}{2\sqrt{2}}\tan(\sqrt{2}\tan x)+C$

25 $\int \frac{dx}{4+5\cos x}\,dx$

Ans. $\frac{1}{3}\log\frac{3+\tan x/2}{3-\tan x/2}+C$

26 $\int \frac{1}{\sin^2 x+\sin 2x}\,dx$

Ans. $\frac{1}{2}\log\frac{\tan x}{\tan x+2}+C$

27 $\int \frac{\sin x}{\sin x-\cos x}\,dx$

Ans. $\frac{1}{2}x+\frac{1}{2}\log(\sin x-\cos x)+C$

28 $\int x^3 e^{x^2}\,dx$

Ans. $\frac{1}{2}e^{x^2}(x^2-1)+C$

29 $\int e^x(\tan x+\sec^2 x)\,dx$

Ans. $\sqrt{2}e^{x/2}\sin(x/2)+C$

30 $\int (\sin(\log x)+\cos(\log x))\,dx$

Ans. $e^x \tan x + C$

31 $\int \sec^3 x\,dx$

Ans. $x \sin \log x + C$

32 $\int \frac{1}{\sin^4 x+\cos^4 x}\,dx$

Ans. $\frac{1}{2}\sec x\tan x+\frac{1}{2}\log(\sec x+\tan x)+C$

33 $\int \frac{e^x(1+x)}{\cos(xe^x)}\,dx$

Ans. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+C$

34 $\int \frac{\sin 3x}{\sin 5x \sin 2x}\,dx$

Ans. $\frac{1}{2}\sqrt{x^2+1}+\frac{1}{2}|\log x^2+\sqrt{x^4+1}|+C$

35 $\int \frac{1}{x^4-81}\,dx$

Ans. $\log|\sec(xe^x)+\tan(xe^x)|+C$

36 $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$

Ans. $\frac{1}{2}\log|\sin 2x|-\frac{1}{5}\log|\sin 5x|+C$

37 $\int_0^\infty \frac{\log x}{1+x^2}\,dx$

Ans. $\frac{1}{18}\left[\frac{1}{6}\log\left|\frac{x-3}{x+3}\right|-\frac{1}{3}\tan^{-1}\frac{x}{3}\right]+C$

38 $\int_{-\pi/2}^{\pi/2}(\sin|x|+\cos|x|)\,dx$

Ans. $\frac{\pi}{8}\log 2$

39 $\int_0^{\pi/2}\frac{\sin x}{1+\cos^2 x}\,dx$

Ans. $\frac{\pi\alpha}{\sin\alpha}$

40 $\int\frac{\sin x+\cos x}{\sqrt{1-\sin x\cos x}}\,dx$

Ans. $\sqrt{2}\log|\sin x-\cos x|+C$

41 $\int\frac{1}{\sec x+\sin x}\,dx$

Ans. $\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x-\cos x}{\sqrt{3}-\sin x+\cos x}\right|+\tan^{-1}(\sin x+\cos x)+C$

42 $\int\frac{1}{\sec x+\operatorname{cosec} x}\,dx$

Ans. $\frac{1}{2}(\sin x-\cos x)-\frac{1}{2\sqrt{2}}\log\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)$

43 $\int\sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}\,dx$

Ans. $\cos\alpha\sin^{-1}\left(\frac{\cos x}{\cos\alpha}-\sin\alpha\log\sin x\right)+\sqrt{\sin^2 x-\sin^2\alpha}+C$

44 $\int\frac{\sqrt{\cos 2x}}{\sin x}\,dx$

Ans. $\frac{1}{2}\log\left|\frac{1+t}{1-t}\right| + \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|$ where $t = \sqrt{1-\tan^2 x}$

45 $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Ans. $(a+x)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + C$

46 $\int \frac{x^2\,dx}{(x\sin x + \cos x)^2}$

Ans. $\frac{\sin x - x\cos x}{x\sin x + \cos x} + C$

47 $\int \frac{\sec x\,dx}{\sqrt{\sin(2x+\alpha) + \sin\alpha}}$

Ans. $\sqrt{2}\ \sqrt{(\tan x + \tan\alpha)\sec\alpha} + C$

48 $\int \frac{\sin x}{\sin 4x}\,dx$

Ans. $\frac{1}{4}\left[\frac{1}{2}\log\frac{1+t}{1-t} + \frac{\sqrt{2}}{2}\log\frac{1-\sqrt{2}t}{1+\sqrt{2}t}\right] + C$

49 $\int \sqrt{\sec x - 1}\,dx$

Ans. $\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}+\sqrt{2x}}{\sqrt{1+x^2}-\sqrt{2x}}\right| - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right| + C$

50 $\int x\sqrt{\frac{1-x}{1+x}}\,dx$

Ans. $\left(\frac{x}{2}-1\right)\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}x + C$

51 $\int \frac{1}{x^4+2x^2+16}\,dx$

Ans. $\frac{1}{8}\left[\frac{1}{\sqrt{10}}\tan^{-1}\frac{x^2-4}{x\sqrt{10}} - \frac{1}{2\sqrt{6}}\log\left|\frac{x^2+4-\sqrt{6}x}{x^2+4+\sqrt{6}x}\right|\right] + C$

52 $\int \frac{1+x^2}{\sqrt{1-x^2}}\,dx$

Ans. $\left[\frac{3}{2}\sin^{-1}\frac{x}{2}\sqrt{1-x^2} + C\right]$

53 $\int \sqrt{\frac{x}{x^3-a^3}}\,dx$

Ans. $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + C$

54 Evaluate : $\int \frac{1+x^2}{1+x^4} dx.$ **(2007)**

55 Evaluate : $\int \cos 4x \cos 3x \, dx.$ **(2007)**

56 Using properties of definite integrals, prove the following : $\int_0^x \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}.$

OR

Evaluate : $\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx.$ **(2007)**

57 Evaluate : $\int \sin 4x \ \sin 8x \ dx$ **(2007 Comp.)**

58 Evaluate : $\int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$ **(2007 Comp.)**

59 Using properties of definite integrals prove the following : $\int_0^2 x\sqrt{2-x} dx = \frac{16}{15}\sqrt{2}$ **(2007 Comp.)**

60 Evaluate : $\int x \cdot \log(x+1) dx$

OR

Using properties of definite integrals evaluate the following: $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ **(2008 Comp.)**

61 Evaluate $\int_1^3 (x^2+5x) dx$ as the limit of sums

OR

Evaluate : $\int \frac{x^2}{x^4+x^2+1} dx$ **(2008 Comp.)**

62 Evaluate : $\int \frac{x^2}{1+x^3} dx.$ **(2008)**

63 Evaluate : $\int_0^1 \frac{dx}{1+x^2}.$ **(2008)**

64 Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ **(2008)**

65 Evaluate : $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx$ **(2008)**

66 Evaluate : $\int \left(\operatorname{cosec}^2 x - \cot x\right) e^x dx$ **(2009 Comp.)**

67 Evaluate : $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx$

OR

Using properties of definite integrals, evaluate the following: $\int_0^{\pi} \frac{x \sin x}{x + \cos^2 x}dx$ **(2009 Comp.)**

68 Evaluate $\int_1^3 \left(2x^2+3\right)dx$ as the limit of sums.

OR

Evaluate $\int \frac{\tan x + \tan^3 x}{1+\tan^3 x}dx$ **(2009 Comp.)**

69 Evaluate : $\int \frac{\sec^2 x}{3+\tan x}dx$ **(2009)**

70 If $\int_0^1 \left(3x^2+2x+k\right)dx = 0,$ find the value of k. **(2009)**

71 Evaluate : $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$ **(2009)**

72 Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$

OR

Evaulate : $\int_0^{\pi/2} \left(2\log\sin x - \log\sin 2x\right)dx.$ **(2009)**

73 Evaluate : $\int \frac{x^3-1}{x^2}dx$ **(2010 Comp.)**

74 Evaluate : $\int_{-\pi/4}^{\pi/4} \sin^3 x dx$ **(2010 Comp.)**

75 Evaluate $\int \left|\log\left(\log x\right) + \frac{1}{\left(\log x\right)^2}\right| dx$

OR

Evaluate : $\int \frac{dx}{(x^2+1)(x^2+2)}$ **(2010 Comp.)**

76

Evaluate $\int_1^2 (x^2+5x)\,dx$ as limit of sums.

OR

Evaluate : $\int (\sqrt{\tan x}+\sqrt{\cot x})\,dx$ **(2010 Comp.)**

77

Evaluate : $\int \frac{\log x}{x}\,dx$ **(2010)**

78

Evaluate : $\int e^x \left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$

OR

Evaluate : $\int \frac{1-x^2}{x(1-2x)}dx$ **(2010)**

79

Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$ **(2010)**

80

Evaluate : $\int \frac{x^3-x^2+x-1}{x-1}\,dx$ **(2011 Comp.)**

81

Evaluate : $\int_0^1 \frac{dx}{1+x^2}$ **(2011 Comp.)**

82

Evaluate : $\int \frac{\sin x-\cos x}{\sqrt{\sin 2x}}dx$

OR

Evaluate : $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ **(2011 Comp.)**

83

Write the value of $\int \sec x(\sec x+\tan x)\,dx$ **(2011)**

84

Write the value of $\int \frac{dx}{x^2+16}$ **(2011)**

85	Evaluate : $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$ OR Evaluate : $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$ **(2011)**
86	Evaluate : $\int_0^{\pi/2} 2\sin x \cos x \tan^{-1}(\sin x)\,dx$ OR Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$ **(2011)**
87	Evaluate the integral : $\int \frac{1}{\sec x - \tan x}\,dx$.
88	Evaluate the integral : $\int \log x\,dx$.
89	Evaluate the integral : $\int_0^1 \frac{2x}{5x^2+1}\,dx$.
90	Evaluate the integral : $\int \frac{e^x}{e^x+1}\,dx$
91	Evaluate the integral : $\int \sin 2x \sin 5x\,dx$
92	Evaluate the integral : $\int_{-\pi}^{\pi} x^{20} \sin^9 x\,dx$.
93	Evaluate the integral : $\int_{-1}^{1} \log\left(\frac{4-x}{4+x}\right)\,dx$.
94	Evaluate the integral : $\int_0^1 e^{2-5x}\,dx$.
95	Evaluate the integral : $\int \frac{2}{3-2x}\,dx$.
96	Evaluate the integral : $\int_1^3 \frac{4}{x}\,dx$.
97	Evaluate the integral : $\int \frac{\sin 2x \cos 2x}{\sqrt{9-\cos^4 2x}}\,dx$

98	Evaluate the integral : $\int \frac{x^2}{\left(x^2+2\right)\left(x^2+3\right)}\,dx$.
99	Evaluate the integral : $\int_2^4 e^x\,dx$, as limit of sums.
100	Evaluate the integral : $\int \frac{(1+\sin x)e^x}{(1+\cos x)}\,dx$.
101	Evaluate the integral : $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\cot^{3/2} x}$.
102	Evaluate the integral : $\int \frac{1}{x^2\left(x^4+1\right)^{3/4}}\,dx$.
103	Evaluate the integral : $\int \frac{\sin x}{\sin(x+\alpha)}\,dx$
104	Evaluate the integral : $\int_0^a \sqrt{\frac{a}{a-x}}\,dx$.
105	Evaluate the integral : $\int e^x\left\{\tan^{-1} x+\frac{1}{1+x^2}\right\}\,dx$.
106	Evaluate the integral : $\int_0^2\left(3x^2+4\right)\,dx$.
107	Evaluate the integral : $\int_0^1 x\left(\tan^{-1} x\right)^2\,dx$.
108	Evaluate the integral : $\int \frac{dx}{x\left[6(\log x)^2+7\log x+2\right]}$
109	Evaluate the integral : $\int_1^4 (\|x-1\|+\|x-2\|+\|x-3\|)\,dx$.
110	Evaluate the integral : $\int_1^2\left(2x^2+x+7\right)\,dx$, as a limit of sums.
111	Evaluate the integral : $\int_{-1}^{3/2} \left|x\sin \pi x\right|\,dx$.
112	Evaluate the integral : $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\,dx$.

113	Evaluate the integral : $\int_2^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\,dx$
114	Evaluate the integral : $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx$.
115	Evaluate the integral : $\int_{0.2}^{3.5} [x]\,dx$.
116	Show that : $\int_0^{\pi} \frac{x \sin x}{1+\sin x}\,dx = \frac{\pi}{2}(\pi - 2)$.